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# KC-6121

## B. E. - II (Sem. III) (EC/EL/CHEM) Examination

**November / December – 2012**

## Engg. Maths - III

Time : **3** Hours] [Total Marks : **100**

**Instructions :**

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
**Fillup strictly the details of ☛ signs on your answer book.**

Name of the Examination :
☛ **B. E. - 2 (Sem. 3) (EC/EL/CHEM)**

Name of the Subject :
☛ **ENGG. MATHS - 3**

☛ Subject Code No. : **6 1 2 1** ☛ Section No. (1, 2,.....) : **1&2**

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figures on right indicate marks.

### SECTION - I

1 (a) Do as directed : **10**

(1) Express the integral $\int_0^4 \int_y^4 \frac{x\,dx\,dy}{x^2+y^2}$, changing the order of integration.

(2) Express $\iint \sin \pi\left(x^2+y^2\right)dxdy$ over the region bounded by the circle $x^2+y^2=1$, by changing to polar co-ordinates.

(3) Find unit normal to the surface $x^2+y^2+z^2=16$, at point $\left(3, 2, \sqrt{3}\right)$.

(4) Define a solenoidal vector. Show that $\overline{V}=z^2\hat{i}+x^2\hat{j}+y^2\hat{k}$ is solenoidal.

(5) Define curl of a vector. find curl of vector $\overline{F}=x^2\hat{i}+y^2\hat{j}+z^2\hat{k}\cdot$

(b) Attempt any **three** : **12**

(1) Evaluate $\iint_S \sqrt{xy - y^2}\, dxdy$, where $S$ is a triangle with vertices (0, 0), (10, 1) and (1, 1).

(2) Calculate the area included between the curve $r = a(\sec\theta + \cos\theta)$ and its asymptote.

(3) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

(4) Find the mass of an elliptic plate $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if the density at point $(x, y)$ on it is $\mu xy$. ($\mu$ is a constant).

**2** (a) Attempt any **two** : **6**

(1) Show that

$$\nabla^2 \left(r^n\right) = n(n+1) r^{n-2}$$

(2) If $\vec{F} = 3xy\,\hat{i} - y^2\,\hat{j}$, Evaluate $\int_C \vec{F} \cdot \vec{dr}$, where $C$ is the arc of the parabola $y = 2x^2$ from $(0, 0)$ to $(1, 2)$.

(3) A vector field is given by $\vec{A} = \left(x^2 + xy^2\right)\hat{i} + \left(y^2 + x^2 y\right)\hat{j}$ show that the field is irrotational and find its scalar potential.

(b) Attempt any **two** : **8**

(1) Apply Green's theorem to prove that the area enclosed by a plane curve $C$, is $\frac{1}{2}\oint_C (x\,dy - y\,dx)$.

(2) Evaluate $\oint_C \overline{F}\cdot\overline{dr}$ where $\overline{F} = y\hat{i} + xz^3\hat{j} - zy^3\hat{k}$, where $C$ is the circle $x^2 + y^2 = 4,\ z = 1.5$, using Stoke's theorem.

(3) Use divergence theorem, to evaluate $\iint_S \overline{F}\cdot\overline{dS}$, where $\overline{F} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and $S$ is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

3 (a) Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. **4**

(b) Attempt any **two** : **10**

(1) Find the Fourier series to represent the function $f(x)$ given by

$$f(x) = x \quad \text{for} \quad 0 \le x \le \pi$$
$$= 2\pi - x \quad \text{for} \quad \pi \le x \le 2\pi$$

Deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$$

(2) Obtain a Fourier series to represent $e^{-ax}$ from $x=-\pi$ to $x=\pi$. Hense derive series for $\frac{\pi}{\sinh \pi}$.

(3) Obtain Fourier series to represent $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval $0<x<2\pi$.

## SECTION - II

4 (a) Do as directed : **10**

(1) Define Beta function. Show that it is symmetric in its arguments.

(2) Define error function, show that $erf(-x)=-erf(x)$.

(3) Find value of $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$.

(4) Write Cauchy-Riemann equations in polar form.

(5) Write statement of Cauchy Integral theorem.

(b) Attempt any **two** : **6**

(1) Show that $\int_0^\infty \frac{x^4}{4^x}dx=\frac{24}{(\log 4)^5}$

(2) Show that $\int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx=\Gamma(n),\ n>0$

(3) Show that $\int_0^2 \left(8-x^3\right)^{-\frac{1}{3}} dx=\frac{1}{3}\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$.

(c) Solve any two : **6**

(1) $yzp - xzq = xy$

(2) $x^2 p + y^2 q = (x+y)^2$

(3) $x^2 (y-z) p + y^2 (z-x) q = z^2 (x-y)$.

**5** Attempt any **two** : **12**

(1) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, where the boundary conditions are $u(0,t) = 0 = u(\ell, t) \quad (t > 0)$ and the initial condition $u(x,0) = x$, $\ell$ being the length of the bar.

(2) A tightly stretched flexible string has its ends fixed at $x = 0$ and $x = \ell$. At time $t = 0$, the string is given a shape defined by $f(x) = \mu x(\ell - x)$ where $\mu$ is a constant and then released. Find the displacement of any point $x$ of the string at any time $t > 0$.

(3) Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ for $0<x<\pi$, $0<y<\pi$, with condition given :

$u(0, y)=u(\pi, y)=u(x, \pi)=0$ and $u(x, 0)=\sin^2 x$.

6 (a) Define on analytic function. If $f(z)$ is an analytic function of $z$, then prove that **4**

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$$

(b) Attempt any two : **6**

(1) Determine the analytic function whose real part is $\log\sqrt{x^2+y^2}$.

(2) Find the bilinear transformation which maps $z=1, i, -1$ onto the points $w=i, 0, -i$.

(3) Show that the image of the hyperbola $x^2-y^2=1$, under the transformation $w=\frac{1}{z}$ is the lemniscate $\rho^2=\cos 2\phi$, where $w=\rho e^{i\phi}$

(c) Attempt any **two** : **6**

(1) Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$ where $C$ is the circle $|z| = 2$.

(2) Evaluate $\oint_C \frac{\sin^2 z}{(z - \pi/6)} dz$ where $C$ is the circle $|z| = 1$.

(3) Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz,$ where $C$ is the circle $|z| = \frac{3}{2}$.